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The relative deformation gradient and its application to cylindrical folds with deformed lineations

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Abstract—The relative deformation gradient is defined and used to compare deformations between any two points of a body. This concept is useful in the geometrical analysis of deformation in natural folds, where only deformed planes and straight lines can be observed. The case of cylindrical folds with deformed lineations is studied.

INTRODUCTION

DEFORMATION in folds has been studied first from a geometrical point of view (Weiss 1959, Ramsay 1960), and then analytically, according to the general theory of deformation (Ramsay 1967, Hobbs 1971). But the deformation of planes and straight lines gives insufficient information to determine the total deformation at each point. For example, any homogeneous deformation before folding cannot be determined. What really appears is the heterogeneous part of the deformation gradient field. Cobbold (1977) superposed perturbations on a regional deformation gradient. The relative deformation gradient defined in this paper is useful to compare deformation between any two points x and y, both belonging to the present configuration of the body. If we observe only surfaces or lines which were initially parallel planes or straight lines, we can recognize (in the present state) infinitesimal vectors $\mathbf{dx}$ and $\mathbf{dy}$ which were equal before deformation. Thus, according to definition (1) given below, we get direct information about the relative deformation gradient. This concept is applied to the case of cylindrical folds with deformed lineations.

DEFINITION

Consider a body undergoing a general transformation f, called the deformation

$$x = fX,$$

where X is the position at the initial time T, and x the position at the final time t of a material point (T and t are arbitrary; e.g. Truesdell & Toupin 1960). Denote by $f'(X)$ the deformation gradient at X, which transforms the infinitesimal vector $\mathbf{dX}$ at X into the corresponding vector $\mathbf{dx}$ at x

$$\mathbf{dx} = f'(X)\,\mathbf{dX}.$$

To compare deformations between two points x and y, with respective initial positions X and Y, we define the deformation gradient at x relative to y as the linear operator $f'(x, y)$ which changes $\mathbf{dy}$ into $\mathbf{dx}$ if the corresponding infinitesimal vectors $\mathbf{dY}$ and $\mathbf{dX}$ in the initial state are equal

$$\mathbf{dx} = f'(x, y)\,\mathbf{dy} \quad \text{if} \quad \mathbf{dX} = \mathbf{dY}. \tag{1}$$

From definitions of $f'(X)$, $f'(Y)$ and $f'(x, y)$, it follows immediately that

$$f'(X) = f'(x, y)f'(Y). \tag{2}$$

Equation (2) is somewhat similar to equation (15) of Cobbold (1977), if Y is considered as a fixed reference point. Equation (2) gives

$$f'(x, y) = f'(X)f'(Y)^{-1}; \tag{3}$$

if rectilinear coordinates are used in the initial and in the final states, the components of $f'(x, y)$ are

$$f'(x, y)_{ij} = \frac{\partial x_i}{\partial X_k}\frac{\partial Y_k}{\partial y_j}. \tag{4}$$

From equation (3), the determinant of $f'(x, y)$ is

$$\det f'(x, y) = \frac{dv}{dw}\left(\frac{dV}{dW}\right)^{-1} \text{(relative volume dilation)} \tag{5}$$

where dV and dW are two elements of volume, respectively at X and Y, and dv and dw the corresponding deformed volumes.

A deformation gradient is classically decomposed into a strain (or stretch) [right $S_r(X)$ or left $S_l(X)$] and rotation $R(X)$

$$f'(X) = R(X)S_r(X) = S_l(X)R(X).$$

The relative deformation gradient may be decomposed

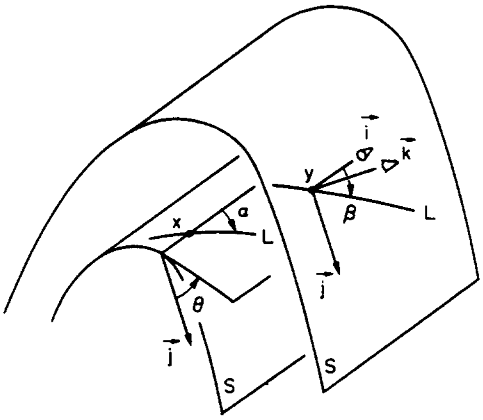

Fig. 1. Cylindrical fold which deforms a set of surfaces (S) and a set of lineations (L). Geometrical parameters and basic vectors.

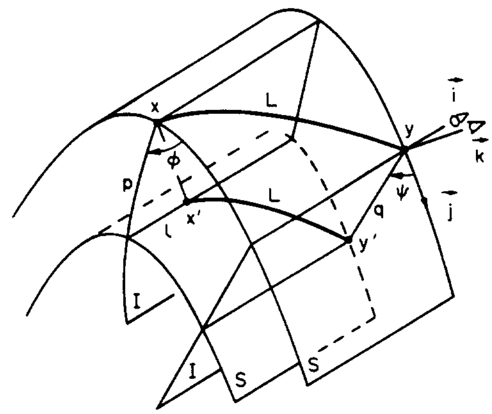

Fig. 2. Geometrical parameters related to isogon surfaces, I. Same notation as in Fig. 1.

in the same way:

$$f'(x, y) = R(x, y)S_r(x, y) = S_l(x, y)R(x, y); \quad (6)$$

$R(x; y)$ is the rotation at x relative to y, $S_r(x, y)$ and $S_l(x, y)$ are the right and left strains at x relative to y.

APPLICATION

Consider a cylindrical fold, which deforms a pre-existing lineation (Fig. 1) and two arbitrary points x and y on the fold. Let **i**, **j** and **k** be three orthogonal unit vectors, with **i** parallel to the fold axis and **j** contained in the plane tangent to the folded surface at y (Fig. 1). Denote by α and β the angles between the fold axis and the lineation at x and at y, respectively, and by θ the angle between the planes tangent to the folded surfaces at x and at y. We suppose that all straight lines parallel to the fold axis have been similarly deformed, which may be written

$$f'(x, y)\mathbf{i} = \mathbf{i}. \quad (7)$$

The vector $\cot \beta\mathbf{i} + \mathbf{j}$, parallel to the lineation at y, is transformed into a vector parallel to the direction $\cot \alpha\mathbf{i} + \cos \theta\mathbf{j} + \sin \theta\mathbf{k}$ of the lineation at x

$$f'(x, y)(\cot \beta\mathbf{i}+\mathbf{j}) = a(\cot \alpha\mathbf{i}+\cos \theta\mathbf{j}+\sin \theta\mathbf{k}),$$

a being a real coefficient. Hence

$$f'(x, y)\mathbf{j} = (a \cot \alpha - \cot \beta)\mathbf{i} + a \cos \theta\mathbf{j} + a \sin \theta\mathbf{k}. \quad (8)$$

Equations (7) and (8) may be written in the matrix form

matrix of $f'(x, y)$

$$= \begin{pmatrix} 1 & a \cot \alpha - \cot \beta & b \\ 0 & a \cos \theta & c \\ 0 & a \sin \theta & d \end{pmatrix}, \quad (9)$$

where a, b, c and d are coefficients depending on x and y. If there is no volume change, we have the supplementary equation:

$$ad \cos \theta - ac \sin \theta = 1. \quad (10)$$

Since α, β and θ can be measured on the fold, equalities (9) and (10) show that only three of the nine coefficients of $f'(x, y)$ remain unknown. Thus, although $f'(X)$ and $f'(Y)$ are generally unknown, the relative deformation gradient $f'(x, y)$ is in great part determined from the fold geometry.

EXAMPLE

The preceding results have been applied to a tight fold with long limb and axis plunging gently southwestwards in ferruginous quartzites at Amili in the Akjoujt area, Mauritania (Bronner & Sougy 1969, Sougy 1969). An early lineation is deformed by the fold and the angle between the fold axis and the lineation changes from 12° (in the long limb) to 115° (in the short limb). In the long limb, the bedding surfaces are parallel planes and the lineations are parallel straight lines: probably, this limb has been homogeneously deformed or is undeformed. We shall analyse deformation throughout the fold as if the long limb were undeformed, which means that we try to determine the deformation gradient $f'(x, y)$ relative to a reference point y on the long limb. The lineations remain approximately in a plane, so that the fold is nearly, but not exactly, of similar type. With the further assumption (justified by the preceding analogy with similar folding) that isogon surfaces were parallel planes before deformation, the relative deformation gradient $f'(x, y)$ may be completely determined as follows. Since y is an arbitrary point on the long limb, it may be chosen on the lineation which contains x. The geometry of the fold determines how the small vector $\mathbf{yy'}$ (see Fig. 2), perpendicular to the fold axis and belonging to the isogon surface at y, is transformed by $f'(x, y)$ into the vector $\mathbf{xx'}$ belonging to the isogon surface at x. With notations of Fig. 2:

$$f'(x, y)(q \cos \psi\mathbf{j} + q \sin \psi\mathbf{k}) = l\mathbf{i} + p \cos (\theta + \varphi)\mathbf{j} + p \sin (\theta + \varphi)\mathbf{k},$$

equivalent, with the help of the matrix (9), to the three equations:

$$(a \cot \alpha - \cot \beta)q \cos \psi + bq \sin \psi = l, \quad (11)$$

$$a \cos \theta q \cos \psi + cq \sin \psi = p \cos(\theta + \varphi), \quad (12)$$

$$a \sin \theta q \cos \psi + dq \sin \psi = p \sin(\theta + \varphi). \quad (13)$$

From equations (10), (12) and (13) we obtain:

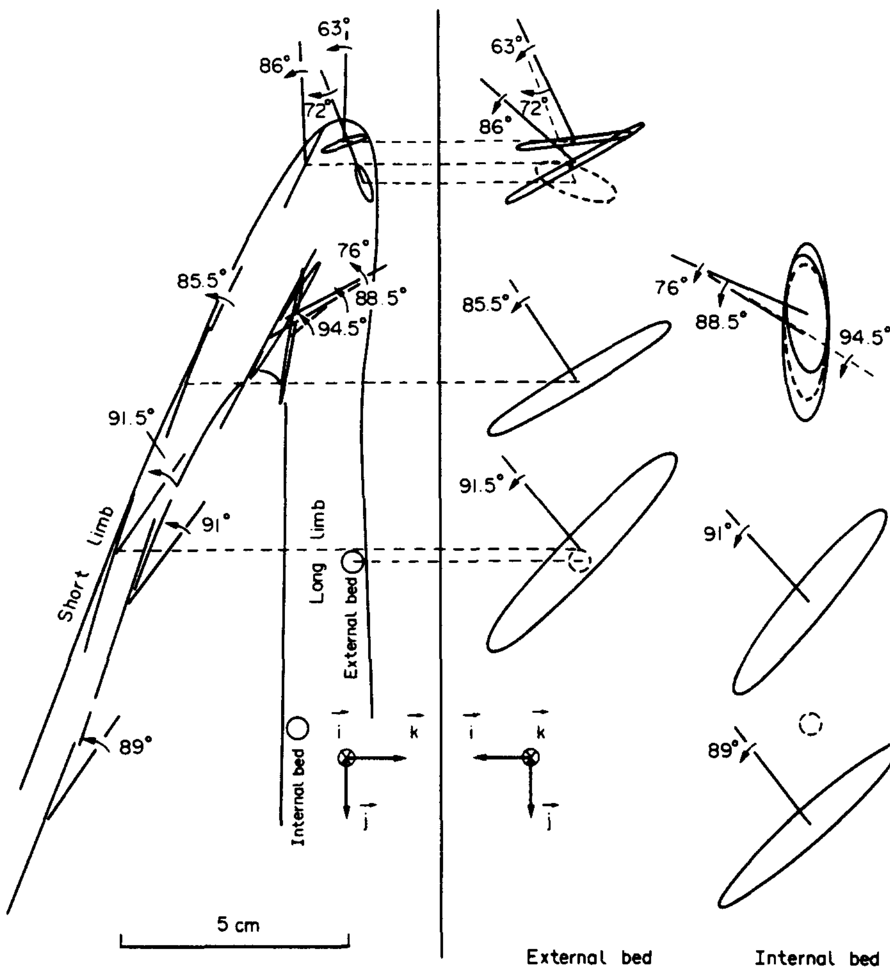

Fig. 3. Field of local rotations and strain ellipsoids, relative to the long limb in the example. Left part: projection along the fold axis; right part: projection perpendicularly to the long limb (rotation axes, strain ellipsoids and spheres situated in the long limb side are represented by dashed lines). In the hinge zone, note the different behaviours of the competent (quartzose) external bed and the incompetent (ferruginous) internal bed.

$$a = \frac{q \sin \psi}{p \sin \varphi}; \tag{14}$$

b, c and d are then respectively deduced from equations (11), (12) and (13). Since l, p, q, φ and ψ can be measured on the fold, the matrix (9) of $f'(x, y)$ is now completely determined. According to equation (6), at any point x on the fold, $f'(x, y)$ may be decomposed into rotation $R(x, y)$ and strain $S_1(x, y)$, which may be geometrically represented by (i) the axis and angle of rotation and (ii) the strain ellipsoid; these are obtained by determining the principal axes and proper numbers of the left (relative) Cauchy tensor:

$$C_1(x, y) = f'(x, y)f'(x, y)^t = S_1(x, y)^2.$$

Results are presented in Fig. 3. According to equation (2), the deformation gradient at any point X is equal to the (unknown) deformation gradient in the long limb multiplied by the determined relative deformation gradient $f'(x, y)$.

REFERENCES

Bronner, G. & Sougy, J. 1969. Etude structurale de la région de Tourarine (Mauritanides, NW d'Akjoujt, République Islamique de Mauritanie), 5ème Colloque Géol. afr., Clermont-Ferrand, 9–11 avril, *Ann. Fac. Sci. Univ. Clermont-Ferrand, Géol. Minéral.*, **41**, 77–78.

Cobbold, P. R. 1977. Description and origin of banded deformation structures—I. Regional strain, local perturbations, and deformation bands. *Can. J. Earth Sci.* **14**, 1721–1731.

Hobbs, B. E. 1971. The analysis of strain in folded layers. *Tectonophysics* **11**, 329–375.

Ramsay, J. G. 1960. The geometry of early linear structures in areas of repeated folding. *J. Geol.* **68**, 75–93.

Ramsay, J. G. 1967. *Folding and Fracturing of Rocks*. McGraw Hill, New York.

Sougy, J. 1969. Grandes lignes structurales de la chaîne des Mauritanides et de son avant pays (socle précambrien et sa couverture infracambrienne et paléozoïque), Afrique de l'Ouest, *Bull. Soc. géol. Fr. 7 Sér.* **11**, 133–149.

Truesdell, C. & Toupin, R. A. 1960. The classical field theories, in: *Handbuch der Physik* (edited by Flügge, S.). Springer, Berlin, Vol. III/I, pp. 226–793.

Weiss, L. E. 1959. Geometry of superposed folding. *Bull. geol. Soc. Am.* **70**, 91–106.